

Table 6.1 Table Laplace transforms

Case	$\overline{F}(s)$	$F(t)$
1	$\frac{1}{s}$	1
2	$\frac{1}{s^2}$	$t$
3	$\frac{1}{s^n} \quad n = 1, 2, 3, \dots$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
5	$s^{-3/2}$	$2\sqrt{t/\pi}$
6	$s^{-(n+1/2)} \quad n = 1, 2, 3, \dots$	$\frac{2^n}{[1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)]\sqrt{\pi}} t^{n-1/2}$
7	$\frac{1}{s^n} (n > 0)$	$\frac{1}{\Gamma(n)} t^{n-1}$
8	$\frac{1}{s \mp a}$	$e^{\pm at}$
9	$\frac{1}{(s+a)^n} \quad n = 1, 2, 3, \dots$	$\frac{t^{n-1} e^{-at}}{(n-1)!}$
10	$\frac{1}{(s+a)(s+b)} \quad (a \neq b)$	$\frac{e^{-at} - e^{-bt}}{b-a}$
11	$\frac{s}{(s+a)(s+b)} \quad (a \neq b)$	$\frac{ae^{-at} - be^{-bt}}{a-b}$
12	$\frac{1}{s^2 + a^2}$	$\frac{1}{a} \sin at$
13	$\frac{s}{s^2 + a^2}$	$\cos at$
14	$\frac{1}{s^2 - a^2}$	$\frac{1}{a} \sinh at$
15	$\frac{s}{s^2 - a^2}$	$\cosh at$
16	$\frac{1}{s(s^2 + a^2)}$	$\frac{1}{a^2} (1 - \cos at)$
17	$\frac{1}{s^2(s^2 + a^2)}$	$\frac{1}{a^3} (at - \sin at)$
18	$\frac{1}{(s^2 + a^2)^2}$	$\frac{1}{2a^3} (\sin at - at \cos at)$
19	$\frac{s}{(s^2 + a^2)^2}$	$\frac{t}{2a} \sin at$
20	$\frac{s^2}{(s^2 + a^2)^2}$	$\frac{1}{2a} (\sin at + at \cos at)$
21	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$	$t \cos at$

Case	$\overline{F}(s)$	$F(t)$
22	$\frac{1}{\sqrt{s+a}}$	$\frac{1}{\sqrt{\pi t}} - ae^{a^2 t} \operatorname{erfc}(a\sqrt{t})$
23	$\frac{\sqrt{s}}{s-a^2}$	$\frac{1}{\sqrt{\pi t}} + ae^{a^2 t} \operatorname{erf}(a\sqrt{t})$
24	$\frac{\sqrt{s}}{s+a^2}$	$\frac{1}{\sqrt{\pi t}} - \frac{2a}{\sqrt{\pi}} e^{-a^2 t} \int_0^{a\sqrt{t}} e^{\lambda^2} d\lambda$
25	$\frac{1}{\sqrt{s(s-a^2)}}$	$\frac{1}{a} e^{a^2 t} \operatorname{erf}(a\sqrt{t})$
26	$\frac{1}{\sqrt{s(s+a^2)}}$	$\frac{2}{a\sqrt{\pi}} e^{-a^2 t} \int_0^{a\sqrt{t}} e^{\lambda^2} d\lambda$
27	$\frac{b^2 - a^2}{(s-a^2)(b+\sqrt{s})}$	$e^{a^2 t} [b - a \operatorname{erf} a\sqrt{t}] - be^{b^2 t} \operatorname{erfc} b\sqrt{t}$
28	$\frac{1}{\sqrt{s}(\sqrt{s+a})}$	$e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$
29	$\frac{1}{(s+a)\sqrt{s+b}}$	$\frac{1}{\sqrt{b-a}} e^{-at} \operatorname{erf}(\sqrt{b-a}\sqrt{t})$
30	$\frac{\sqrt{s+2a}}{\sqrt{s}} - 1$	$ae^{-at} [I_1(at) + I_0(at)]$
31	$\frac{1}{\sqrt{s+a}\sqrt{s+b}}$	$e^{-(a+b)t/2} I_0\left(\frac{a-b}{2}t\right)$
32	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
33	$\frac{(\sqrt{s^2+a^2}-s)^v}{\sqrt{s^2+a^2}} \quad (v > -1)$	$a^v J_v(at)$
34	$\frac{s}{(s^2+a^2)^{3/2}}$	$tJ_0(at)$
35	$\frac{1}{(s^2+a^2)^{3/2}}$	$\frac{tJ_1(at)}{a}$
36	$\frac{s}{(s^2-a^2)^{3/2}}$	$tI_0(at)$
37	$\frac{1}{(s^2-a^2)^{3/2}}$	$\frac{tI_1(at)}{a}$
38	$\frac{(s-\sqrt{s^2+a^2})^v}{\sqrt{s^2+a^2}} \quad (v > -1)$	$a^v I_v(at)$
39	$\frac{1}{s} e^{-ks}$	$U(t-k)$ [ $U(t) \equiv$ unit-step function]

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Case	$\overline{F}(s)$	$F(t)$
40	$\frac{1}{s^2} e^{-ks}$	$(t - k) U(t - k)$
41	1	$\delta(t)$ [ $\delta(t) \equiv$ delta function]
42	$e^{-as}$	$\delta(t - a)$
43	$\frac{1}{s} e^{-k/s}$	$J_0(2\sqrt{kt})$
44	$\frac{1}{s^\mu} e^{-k/s} \quad (\mu > 0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}(2\sqrt{kt})$
45	$\frac{1}{s^\mu} e^{k/s} \quad (\mu > 0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}(2\sqrt{kt})$
46	$e^{-k\sqrt{s}} \quad (k > 0)$	$\frac{k}{2\sqrt{\pi t^3}} \exp\left(-\frac{k^2}{4t}\right)$
47	$\frac{1 - e^{-k\sqrt{s}}}{s} \quad (k \geq 0)$	$\operatorname{erf}\left(\frac{k}{2\sqrt{t}}\right)$
48	$\frac{1}{s} e^{-k\sqrt{s}} \quad (k \geq 0)$	$\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
49	$\frac{1}{\sqrt{s}} e^{-k\sqrt{s}} \quad (k \geq 0)$	$\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$
50	$\frac{1}{s^{3/2}} e^{-k\sqrt{s}} \quad (k \geq 0)$	$2\sqrt{\frac{t}{\pi}} \exp\left(-\frac{k^2}{4t}\right) - k \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
51	$\frac{1}{s^{1+n/2}} e^{-k\sqrt{s}} \quad (n = 0, 1, 2, \dots, k \geq 0)$	$(4t)^{n/2} \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
52	$\frac{e^{-k\sqrt{s}}}{a + \sqrt{s}} \quad (k \geq 0)$	$\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{k^2}{4t}\right)$ $- a e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$
53	$\frac{e^{-k\sqrt{s}}}{\sqrt{s}(a + \sqrt{s})} \quad (k \geq 0)$	$e^{ak} e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right)$
54	$\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}} \quad (k \geq 0)$	$e^{-at/2} I_0\left(\frac{1}{2} a \sqrt{t^2 - k^2}\right) U(t - k)$
55	$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}} \quad (k \geq 0)$	$J_0\left(a\sqrt{t^2 - k^2}\right) U(t - k)$
56	$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2 - a^2}} \quad (k \geq 0)$	$I_0\left(a\sqrt{t^2 - k^2}\right) U(t - k)$

Case	$\bar{F}(s)$	$F(t)$
57	$\frac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})} \quad (k \geq 0)$	$-e^{ak}e^{a^2t} \operatorname{erfc}\left(a\sqrt{t} + \frac{k}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
58	$\frac{1}{s^2}e^{-k\sqrt{s}}$	$\left(t + \frac{k^2}{2}\right) \operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right) - k\left(\frac{t}{\pi}\right)^{1/2} \exp\left(-\frac{k^2}{4t}\right)$
59	$\frac{1}{s} \ln s$	$-\gamma - \ln t$
60	$-\frac{(\gamma + \ln s)}{s}$	$[\gamma \equiv \text{Euler's constant}]$ $\ln t$
61	$\ln \frac{s+a}{s+b}$	$\frac{1}{t}(e^{-bt} - e^{-at})$
62	$\ln \frac{s^2+a^2}{s^2}$	$\frac{2}{t}(1 - \cos at)$
63	$\ln \frac{s^2-a^2}{s^2}$	$\frac{2}{t}(1 - \cosh at)$
64	$K_0(ks) \quad (k > 0)$	$\frac{1}{\sqrt{t^2-k^2}}U(t-k)$
65	$K_0(k\sqrt{s}) \quad (k > 0)$	$\frac{1}{2t} \exp\left(-\frac{k^2}{4t}\right)$
66	$\frac{1}{\sqrt{s}}K_1(k\sqrt{s}) \quad (k > 0)$	$\frac{1}{k} \exp\left(-\frac{k^2}{4t}\right)$
67	$\frac{\sinh xs}{s \sinh as}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \lambda_n x \cos \lambda_n t \quad \left[\lambda_n \equiv \frac{n\pi}{a}\right]$
68	$\frac{\sinh xs}{s \cosh as}$	$\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} \sin \lambda_n x \sin \lambda_n t$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
69	$\frac{\cosh xs}{s \sinh as}$	$\frac{t}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos \lambda_n x \sin \lambda_n t$ with $\lambda_n \equiv \frac{n\pi}{a}$

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Case	$\bar{F}(s)$	$F(t)$
70	$\frac{\cosh xs}{s \cosh as}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} \cos \lambda_n x \cos \lambda_n t$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
71	$\frac{\sinh xs}{s^2 \sinh as}$	$\frac{xt}{a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \sin \lambda_n x \sin \lambda_n t$ with $\lambda_n \equiv \frac{n\pi}{a}$
72	$\frac{\sinh xs}{s^2 \cosh as}$	$x + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \sin \lambda_n x \cos \lambda_n t$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
73	$\frac{\cosh xs}{s^2 \sinh as}$	$\frac{t^2}{2a} + \frac{2a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \lambda_n x (1 - \cos \lambda_n t)$ with $\lambda_n \equiv \frac{n\pi}{a}$
74	$\frac{\cosh xs}{s^2 \cosh as}$	$t + \frac{8a}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \cos \lambda_n x \sin \lambda_n t$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
75	$\frac{\cosh xs}{s^3 \cosh as}$	$\frac{t^2 + x^2 - a^2}{2} - \frac{16a^2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos \lambda_n x \cos \lambda_n t$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
76	$\frac{\sinh x \sqrt{s}}{\cosh a \sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^n \lambda_n \sin \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{n\pi}{a}$
77	$\frac{\cosh x \sqrt{s}}{\sinh a \sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n-1} \lambda_n \cos \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$

Case	$\bar{F}(s)$	$F(t)$
78	$\frac{\sinh x \sqrt{s}}{\sqrt{s} \cosh a \sqrt{s}}$	$\frac{2}{a} \sum_{n=1}^{\infty} (-1)^{n-1} \sin \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
79	$\frac{\cosh x \sqrt{s}}{\sqrt{s} \sinh a \sqrt{s}}$	$\frac{1}{a} + \frac{2}{a} \sum_{n=1}^{\infty} (-1)^n \cos \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{n\pi}{a}$
80	$\frac{\sinh x \sqrt{s}}{s \sinh a \sqrt{s}}$	$\frac{x}{a} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{n\pi}{a}$
81	$\frac{\cosh x \sqrt{s}}{s \cosh a \sqrt{s}}$	$1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} \cos \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$
82	$\frac{\sinh x \sqrt{s}}{s^2 \sinh a \sqrt{s}}$	$\frac{xt}{a} + \frac{2}{a} \sum_{n=1}^{\infty} \frac{(-1)^n}{\lambda_n^3} \sin \lambda_n x (1 - e^{-\lambda_n^2 t})$ with $\lambda_n \equiv \frac{n\pi}{a}$
83	$\frac{\cosh x \sqrt{s}}{s^2 \cosh a \sqrt{s}}$	$\frac{x^2 - a^2}{2} + t - \frac{2}{a} \sum_{n=1}^{\infty} \frac{(-1)^n}{\lambda_n^3} \cos \lambda_n x e^{-\lambda_n^2 t}$ with $\lambda_n \equiv \frac{(2n-1)\pi}{2a}$